Effect of colored noises on spatiotemporal chaos in the complex Ginzburg-Landau equation

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The effect of colored noises, which are correlated both in space and time, on spatiotemporal chaos in the complex Ginzburg-Landau equation has been studied numerically. The correlations of spatiotemporal patterns as a function of characteristics of noise were calculated. We found that there exists an optimal correlation length of noise where the system establishes its maximal spatial correlation; a small temporal correlation of noise corresponds to a larger correlation length in the system; and that an increase of noise intensity enhances the spatial correlation of the spatiotemporal patterns. Besides, the frequency of temporal correlation function, which is a complicated oscillation, also depends on properties of the noise.

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The nontrivial role played by noise in dynamic systems has been recognized for over two decades [1,2]. Due to its theoretical importance and potential practical application, it has attracted great interest of theoretical and experimental researches. Earlier investigations along this line were mainly concentrated on low-dimensional systems with purely temporal dependence. Important effects have been discovered. It was found that noise can change the instability of deterministic states, and induce new states that are completely absent in the noise-free systems. As a usual source of disorder, it can counterintuitively produce order. Under appropriate circumstances, the usual nuisance can become a boon as has been seen in the well-known case of stochastic resonance [3].

As the influence of noises on low-dimensional dynamic systems has been studied extensively [1], much research interest has nowadays shifted to spatially extended systems [4,5], a situation that is apparently much more complicated. In this case, the noise can correlate in both space and time [6], and the influence is even more profound. Important manifestations include noise-induced fronts [7] and spatial patterns [8,9], noise-induced phase transitions (including noise-induced ordering transitions and noise-induced disordering transitions) [10–14], noise-induced phase separation [15], spatiotemporal stochastic resonance [16–20], and various noise-sustained phenomena [21–28]. In the spatially extended situations, the way in which the noise takes effect is not obvious, and the deterministic description usually cannot give the right results. It is assumed that the noise-induced phenomena have come about as a consequence of nonlinear interaction between the noise and the deterministic dynamics.

In this paper, we are concerned with the interplay between colored noises and spatiotemporal chaos in the complex Ginzburg-Landau equation. It is well known that noise and chaos represent, respectively, two kinds of essentially different irregularities. The former is induced by genuine stochastic sources, while the randomness of the latter is pseudo and is deterministic in its origin. The noises we consider are colored, i.e., correlated both spatially and temporally. The spatiotemporal chaos is intrinsically irregular in both space and time, and represents a prototype of deterministic randomness. It is interesting to see what would come about as a result of the interaction between these two irregularities that are essentially distinct.

We fix the deterministic dynamics of complex Ginzburg-Landau equation in the regime of chaos, and change the property of colored noise. The response of the system to the noise is then simulated. Spatial and temporal correlation functions are used to characterize the system. We show that the noise significantly affects the correlation properties of the system. The spatial correlation length and oscillation frequency of the temporal correlation function depend remarkably on the character of noise. We find that there exists an optimal correlation length of the noise where the system achieves its maximum spatial correlation. The correlation time and the intensity of the noise are also found to have significant influence.

The complex Ginzburg-Landau equation (CGLE) is an important model for spatially extended chaos. It is simple, experimentally relevant, universal [29], and has been a generic amplitude equation widely used in the study of pattern formation. We consider the CGLE of the form

$$\frac{\partial A}{\partial t}=A+(1+ic_1)\nabla^2A-(1-ic_3)|A|^2A, \tag{1}$$

where amplitude A is complex, and c_1,c_3 are real positive numbers. The equation supports fruitful spatiotemporal phenomena, and chaos appears as one of its fundamental solutions. A basic source of chaos in the CGLE is the Benjamin-Feir instability, which is an interplay between spatial and temporal dispersion. For our purpose, we fix c_1 and c_3 beyond the Benjamin-Feir curve ($c_1c_3=1$) to ensure a chaotic dynamics. The colored noise we considered has been generated from the following stochastic partial differential equation [30]:

$$\frac{\partial \eta(r,t)}{\partial t}=-\frac{1}{\tau}(1-\lambda^2\nabla^2)\eta+\frac{1}{\tau}\xi(r,t). \tag{2}$$

$\xi(r,t)$ is a Gaussian distributed white noise that has property

$$\langle \xi(r,t)\xi(r',t')\rangle=2\varepsilon\,\delta(t-t')\,\delta(r-r'), \tag{3}$$

where ε is the intensity of the noise. λ in Eq. (2) measures the correlation length of $\eta(r,t)$, and the temporal memory of the stochastic process is controlled by τ. The spatial correlation of order λ is ensured by the Laplacian term that couples the stochastic field at different points. Obviously, Eq. (3) is a generalization of the evolution equation for Ornstein-Uhlenbeck process. The noises determined by Eq. (2) are exponentially correlated both in space and time. The noise generated in this way represents a simple spatiotemporal structured noise that can be used to mimic real situations. Equation (2) is a linear stochastic partial differential equation. By applying the algorithm developed in Ref. [30], we simulate exactly the noise in Fourier space and perform a reverse-Fourier transform back to the real space. In this way we obtain the final noise we need. The colored noise governed by Eq. (2) has been introduced additively into Eq. (1), and we concentrate our attention on the noise-affected CGLE of the following form:

$$\frac{\partial A}{\partial t}=A+(1+ic_1)\nabla^2 A-(1-ic_3)|A|^2 A+(1+i)\eta(r,t). \tag{4}$$

In order to examine the influence of added noise and its interaction with the spatiotemporal chaos, Eq. (4) is integrated numerically. We discreted Eq. (4) on a 512×512 square lattice with a zero-flux boundary condition and integrated it numerically by Euler algorithm. In all our calculations, the parameter c_1,c_3 was fixed to be 0.9 and 1.25, respectively. The dynamics of the noise-free CGLE is thus located in the chaotic regime.

A snapshot of the spatiotemporal chaos is depicted in Fig. 1(a), which is a gray-scaled picture of the real part of the complex amplitude. In order to characterize the system, we introduce the following spatial correlation function:

$$C_s(r)=\left\langle \int \mathrm{Re}(A(r'))\mathrm{Re}(A(r'+r))dr' \right\rangle. \tag{5}$$

It is defined in the field of the real part of the amplitude and measures the spatial correlation property of points apart from distance r. The average $\langle\rangle$ is performed over the time. The integration is carried out numerically over the lattice in a long period of time so that the observed patterns are asymptotic. Figure 2 (open circles) shows the C_s for the noise-free pattern in Fig. 1(a). It is roughly an exponential decay. The solid line over the circles is a least-square fit of exponential function $e^{-r/l}$. The distance where $C_s(r)$ decays to its e^{-1} thus measures the correlation length l, which is 5.25 for the present case.

The behavior of the system undergoes a drastic change when the noise is turned on. Figure 1(b) shows a typical snapshot of the real part of the amplitude after transient process has died out. Large clusters can be recognized in the demonstration. The details of the spatiotemporal chaos [Fig. 1(a)] are smeared out by noises, and small turbulent waves merge to form large domains. This suggests that the spatial correlation of the system is larger than the case when the noise is absent. This is convinced clearly by the correlation function depicted in Fig. 2 (open squares). An exponential line best fits the simulation (solid curve). The decay rate is, however, much slower. The correlation length was fitted to be 10.92, about twice as much as in the noise-free system (5.25). At this point, the colored noise has a correlation length of 5.06 ($\lambda=16$). It is interesting that the interplay between the noise and extended chaos with short spatial correlations was able to establish a much longer spatial correlation within the system. The typical pattern shown in Fig. 1(b) is not static. It evolves continuously with time.

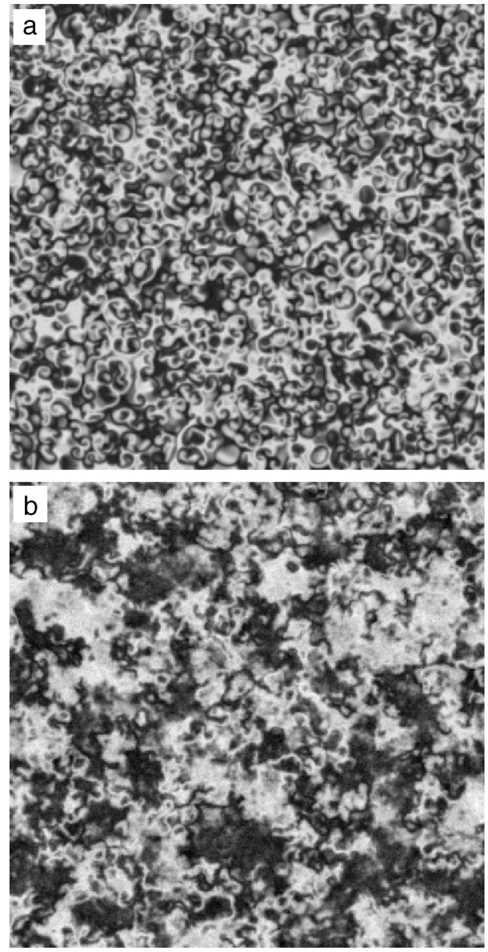

FIG. 1. Snapshot of spatiotemporal chaos (a) before and after the noise is turned on (b). Parameters: $c_1=0.9$, $c_3=1.25$; $\tau=1.0$, $\lambda=16.0$, $\varepsilon=0.001$.

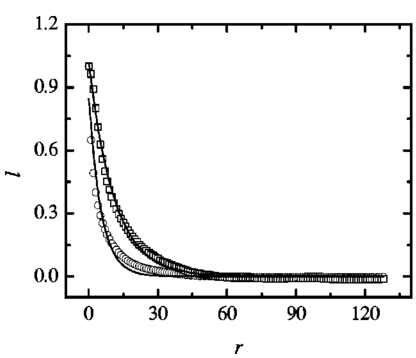

FIG. 2. Normalized correlation function C_s against space r for the noise-free (circle) and noise-affected system (square). The correlation length was fitted exponentially to be about 5.25 and 10.92, respectively. Parameters are the same as in Fig. 1.

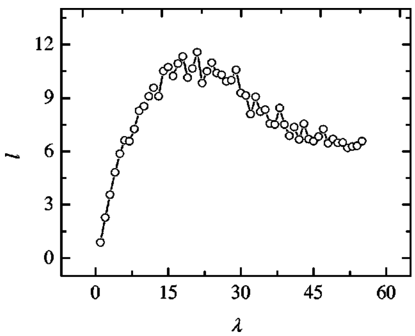

FIG. 3. Spatial correlation length l as a function of λ, showing that at the optimal value $\lambda=21$ the system achieves its maximal correlation length $l=11.57$. Other parameters are the same as in Fig. 1.

In order to obtain a more complete picture of the correspondence between the correlation length of noise and response of the system, we fixed $\tau=1.0$ and scanned the parameter λ in Eq. (2) from 1.0 to 55.0. Figure 3 demonstrated the correlation lengths of patterns against parameter λ. One observed that the spatial correlation of the system does not always go up monotonically with the increase of correlation length of noise. A larger value of λ does not necessarily induce a longer correlation length. There exists an optimal value of λ (21.0) where the spatial correlation of the system achieves its maximum (11.57). This phenomenon is interesting and counterintuitive, indicating a complex interplay between the noise and deterministic chaos.

The parameter τ that controls the temporal correlation of the noise has also significant influence on the spatial correlation of the system. We find that noises with longer time memory produce patterns with a shorter correlation length. We have fixed $\lambda=16.0$ and scanned τ from 1.0 to 40.0. Figure 4 summarizes the simulation results. It is clear that correlation length of the system decreases monotonically with the increase of noise memory. Noises with short correlation time can drastically enhance the spatial correlation. A platform is reached as τ grows up. Notice that the height of the platform is about the value of the correlation length of the noise-free spatiotemporal chaos, which is about 5.25. This suggests that the noise of a specific intensity with a large enough temporal memory no long affects much the spatial correlation of the system.

In the above calculations, the results were obtained with a fixed noise intensity ($\varepsilon=0.001$). When the noise is more intensive, the influence on the spatial correlation is found to be much more enhanced. Figure 5 depicts such an example. The portrait in Fig. 5(a) is a snapshot of the real part of the complex amplitude after the noise has been turned on, with parameter $\varepsilon=0.01$, $\lambda=40.0$, and $\tau=1.0$. Figure 5(b) shows the pattern with a much more intensive noise $\varepsilon=0.1$. It is remarkable that the clusters in Fig. 5(b) is much magnified, indicating a much longer spatial correlation length than that of Fig. 5(a).

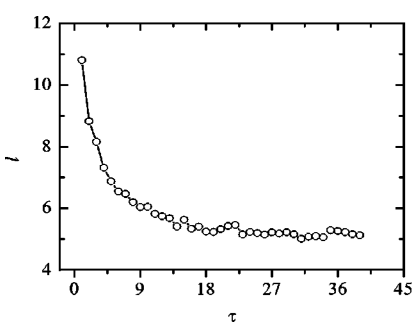

FIG. 4. The correlation length of the noise-affected system as a function of τ when $\varepsilon=0.001$, $\lambda=16.0$.

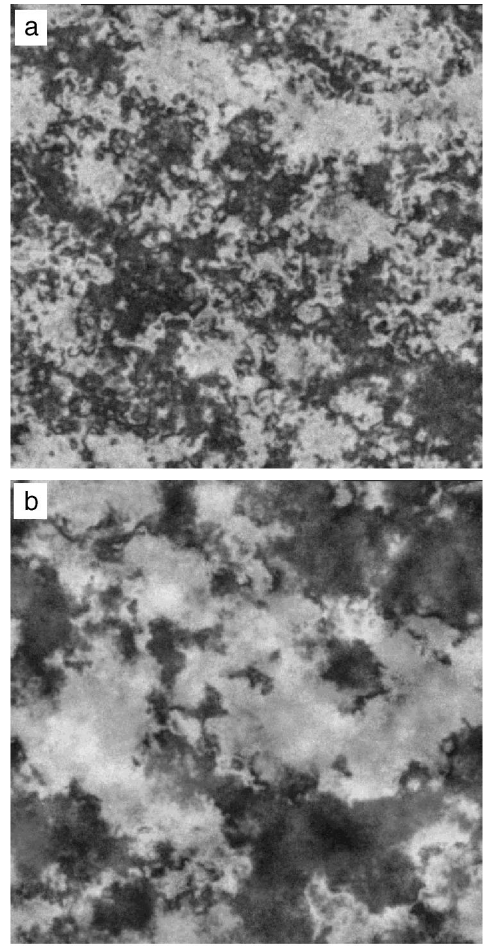

FIG. 5. Patterns with different intensity of noises: $\varepsilon=0.01$ for (a), and $\varepsilon=0.1$ for (b). Other parameters are identical with Fig. 1 except for $\lambda=40$.

We also checked the influence of noise on temporal correlation of the dynamics dictated by Eq. (4). For simplicity, the temporal correlation function was defined at a single point in the real field as follows:

$$C_t(t)=\int \mathrm{Re}(A(t))\mathrm{Re}(A(t'+t))dt'. \tag{6}$$

The correlation functions shown in Fig. 6 were calculated at an arbitrarily chosen point on the lattice (C_t at different points in the field were found to differ trivially). One can see that the functions are no longer simple exponential decays, but curves of complicated oscillations. Figure 6 showed two

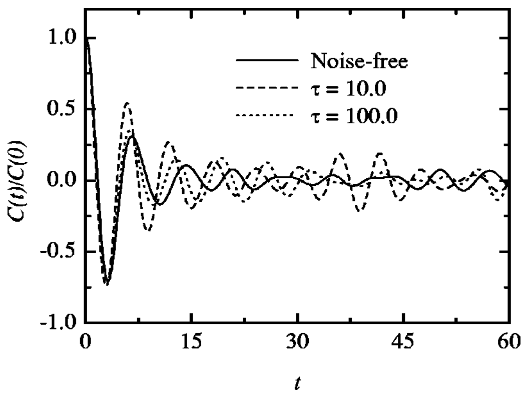

FIG. 6. Damping temporal correlation function C_t when the system is free of noise (solid line), and noise affected with $\tau=10.0$ (dashed line) and $\tau=100.0$ (dotted line). Other parameters are the same as in Fig. 1.

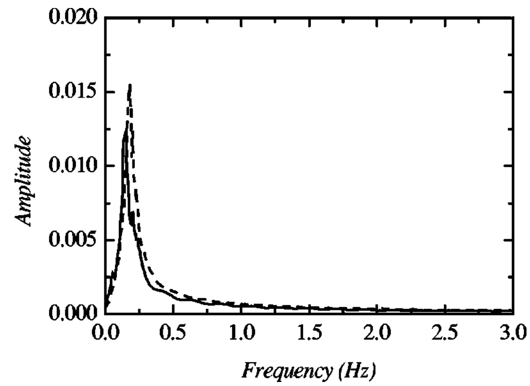

FIG. 7. Fourier transforms of two C_t functions in Fig. 6, i.e., the solid and dashed curves, showing a small frequency shift.

cases with noise $\tau=10.0$ (dashed line) and $\tau=100.0$ (dotted line). The solid curve denotes the noise-free C_t. The effect of the noises on the oscillation frequency of $C_t(t)$ is obvious. As demonstrated in Fig. 7, Fourier transform of the temporal correlation functions clearly shows a small shift in the frequency.

We noticed that the amplitude of the oscillation, in both noise-free and noise-affected situations, is not always damped. The behavior is complicated due to the chaotic nature of the whole dynamics. It decays fast only at the earliest stage. At later time, it typically grows up and then falls once in a while in an irregular manner.

In summary, the interplay between two essentially distinct irregularities, i.e., spatiotemporal colored noises and spatially extended chaos in the complex Ginzburg-Landau equation, has been simulated numerically. We calculated the spatial and temporal correlation of the noise-affected system as functions of the noise, and revealed a complex interaction between chaos and noise. It should be pointed out that the noise we considered is simply additive. Situations where the noise enters the dynamics multiplicatively are also interesting, and potential complex behaviors are possible. In our simulations, we have been only involved in the property of correlation function of the system. Manifestations of noise on other characteristics of spatiotemporal chaos, such as Lyapunov exponents and dimensions, have not been considered. Results presented here provided a first step to explore the possibilities of complex dynamics coming out from the interaction between chaos and noise. Further investigation along this line is desirable.

Compared with the effects of noise on low-dimensional chaos [2,31], the action of structured noise on spatially extended chaos is more significant. In the case of simple chaos, chaotic attractor is robust, and noises have been found to have no drastic influence on the strange attractor. It only smeared its fine structures and made it a little fuzzy. Changes in characteristics of the chaotic dynamics have not been predicted in both simulations or observed in experiments. Our results presented here showed, however, that the impact of colored noise on spatially extended chaos is profound.

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[1] W. Horsthemke and R. Lefever, *Noise-Induced Transitions* (Springer, Berlin, 1984).
[2] K. Lindenberg, B. J. West, and J. Masoliver, *Noise in Nonlinear Dynamical Systems* (Cambridge University Press, Cambridge, 1989).
[3] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).
[4] J. García-Ojalvo and J.M. Sancho, *Noise in Spatially Extended Systems* (Springer, New York, 1999).
[5] A.S. Mikhailov, Phys. Lett. **73A**, 143 (1979); Phys. Rep. **184**, 307 (1989).
[6] J. García-Ojalvo and J.M. Sancho, Phys. Rev. E **49**, 2769 (1994).
[7] M.A. Santos and J.M. Sancho, Phys. Rev. E **59**, 98 (1999).
[8] J. García-Ojalvo, J.M. Sancho, and L. Ramírez-Piscina, Phys. Lett. A **168**, 35 (1992).
[9] J.M.R. Parrondo, C. Van den Broeck, J. Buceta, and J. de la Rubia, Physica A **224**, 153 (1996).
[10] C. Van den Broeck, J.M.R. Parrondo, and R. Toral, Phys. Rev. Lett. **73**, 3395 (1994).
[11] A. Becker and L. Kramer, Phys. Rev. Lett. **73**, 955 (1994).
[12] C. Van den Broeck, J.M.R. Parrondo, J. Armero, and A. Hernández-Machado, Phys. Rev. E **49**, 2639 (1994).
[13] S. Kim, S.H. Park, and C.S. Ryu, Phys. Rev. Lett. **78**, 1616 (1997).
[14] R. Müller, K. Lippert, A. Kühnel, and U. Behn, Phys. Rev. E **56**, 2658 (1997).
[15] J. García-Ojalvo, A.M. Lacasta, J.M. Sancho, and R. Toral, Europhys. Lett. **42**, 125 (1998).
[16] P. Jung and G. Mayer-Kress, Phys. Rev. Lett. **74**, 2130 (1995).
[17] H.S. Wio, Phys. Rev. E **54**, R3075 (1996).
[18] J.M. Vilar and J.M. Bubí, Phys. Rev. Lett. **78**, 2886 (1997).
[19] S. Kim, S.H. Park, and H.-B. Pyo, Phys. Rev. Lett. **82**, 1620 (1999).

[20] B.J. Kim, M. Choi, P. Minnhagen, G.S. Jeon, H.J. Kim, and M.Y. Choi, Phys. Rev. B **63**, 104506 (2001).
[21] S. Kádár, J. Wang, and K. Showalter, Nature (London) **391**, 770 (1998).
[22] J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Locher, and W.L. Ditto, Phys. Rev. Lett. **81**, 5048 (1998).
[23] A. Neiman, L. Schimansky-Geier, A. Cornell-Bell, and F. Moss, Phys. Rev. Lett. **83**, 4896 (1999).
[24] I. Sendiña-Nadal, S. Alonso, V. Pérez-Muñuzuri, M. Gómez-Gesteira, V. Pérez-Villar, L. Ramírez-Piscina, J. Casademunt, J.M. Sancho, and F. Sagués, Phys. Rev. Lett. **84**, 2734 (2000).
[25] V. Pérez-Muñuzuri, F. Sagués, and J.M. Sancho, Phys. Rev. E **62**, 94 (2000).
[26] M. Löcher, N. Chatterjee, F. Marchesoni, W.L. Ditto, and E.R. Hunt, Phys. Rev. E **61**, 4954 (2000).
[27] K.L. Babcock, G. Ahlers, and D.S. Cannell, Phys. Rev. Lett. **67**, 3388 (1991).
[28] S. Alonso, I. Sendiña-Nadal, V. Pérez-Muñuzuri, J.M. Sancho, and F. Sagués, Phys. Rev. Lett. **87**, 078302 (2001).
[29] M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[30] J. García-Ojalvo, J.M. Sancho, and L. Ramírez-Piscina, Phys. Rev. A **46**, 4670 (1992).
[31] J.P. Crutchfield, J.D. Farmer, and B.A. Huberman, Phys. Rep. **92**, 45 (1982).